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DYNAMIC EDGE EFFECTS IN ORTHOTROPIC ELASTIC SHELLS

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A development of the theory of the dynamic edge effect [1, 2] applied to the free vibrations of thin elastic orthotropic shells is given. It is assumed that the lines of curvature, the principal directions of elasticity and the lines along which the boundary conditions are given all coincide. The properties of the characteristic roots of the equations governing the kind of damping rate of the edge effects are investigated. Necessary and sufficient conditions are established for nondegeneration of the edge effects as a function of relationships between the principal curvatures and the shell elasticity coefficients.

1. Let us use the asymptotic method [1, 2] to evaluate the free vibrations frequencies of a thin elastic orthotropic shell whose principal directions of the material elasticity coincide with the coordinate lines x_1, x_2, which are the lines of principal curvatures. We assume compliance with the conditions for applicability of equations of Vlasov type and let us write the dimensionless equations for the preeminently bending vibrations modes as [3]

$$\nabla^4{}_d v + \frac{\partial^2\psi}{\partial\xi_1^2} + \chi\frac{\partial^2\psi}{\partial\xi_2^2} - \omega^2 v = 0 \tag{1.1}$$

$$\nabla_b{}^4\psi - \frac{\partial^2 v}{\partial\xi_1^2} - \chi\frac{\partial^2\psi}{\partial\xi_2^2} = 0$$

$$\nabla_d{}^4 = d_{11}\frac{\partial^4}{\partial\xi_1^4} + 2d_{12}\frac{\partial^4}{\partial\xi_1^2\partial\xi_2^2} + d_{11}^{-1}\frac{\partial^4}{\partial\xi_2^4}$$

$$\nabla_b^4 = \frac{\partial^4}{\partial \xi_1^4} + 2b_{12}\frac{\partial^4}{\partial \xi_1^2 \partial \xi_2^2} + b_{11}\frac{\partial^4}{\partial \xi_2^4}$$

Here we have introduced the following dimensionless variables and parameters

$\xi_1 = x_1 k_0$, $\xi_2 = x_2 k_0$, $v = wk_0$, $\psi = \varphi B_{22} R_2 k_0^3$, $\omega = \Omega / \Omega_2$, $\chi = R_2 / R_1$, $d_{11} = (D_{11} / D_{22})^{1/2}$, $d_{12} = D_{12} / (D_{11} D_{22})^{1/2} = \nu_1 / d_{11} + 2\gamma (1 - \nu_1^2 / d_{11}^2)$, $b_{11} = B_{11} / B_{22}$, $b_{12} = B_{12} / B_{22} = 1 / (2\gamma d_{11}) - \nu_1 / d_{11}^2$, $\gamma = G / (E_1 E_2)^{1/2}$, $k_0 = B_{22}^{-1/4} R_2^{-1/2} (D_{11} D_{22})^{-1/8}$, $\Omega_2 = (\rho h B_{22} R_2^2)^{-1/2}$

where R_1, R_2 are the principal radii of curvatures, w, φ are the normal deflection and force function in the middle surface, respectively, ρh is the surface mass density, Ω is the free vibrations frequency, D_{jk} and B_{jk} are matrix elements of the cylindrical stiffness and the compliance under membrane deformation, E_1, E_2, ν_1, ν_2 ($E_2\nu_1 = E_1\nu_2$), G are the elastic constants of the shell material. The line with least curvature in absolute value is taken as x_1 and hence $|\chi| \leqslant 1$.

We seek the solution of (1.1) in the domain distant from the shell edges in the form

$$v = V_1 \sin k_1 (\xi_1 - \xi_1^\circ) \sin k_2 (\xi_2 - \xi_2^\circ), \qquad (1.2)$$
$$\psi = \Psi_1 \sin k_1 (\xi_1 - \xi_1^\circ) \sin k_2 (\xi_2 - \xi_2^\circ)$$

where k_1, k_2 are dimensionless wave numbers, ξ_1°, ξ_2° are phase constants. Then the asymptotic formula for the free vibrations frequencies of the shell

$$\omega^2 \approx d_{11}k_1^4 + 2d_{12}k_1^2k_2^2 + d_{11}^{-1}k_2^4 + (k_1^2 + \chi k_2^2)[k_1^4 + 2b_{12}k_1^2k_2^2 + b_{11}k_2^4]^{-1} \qquad (1.3)$$

is easily obtained from (1.1) and (1.2). The corresponding wave numbers in the case of nondegeneration of the dynamic edge effect are found from the joint equations according to the procedure of the asymptotic method [1].

We investigate the condition for degeneration of the dynamic edge effect in orthotropic shells. Let us first consider the edge $\xi_1 = 0$. Near the edge the generating solution (1.2) should be corrected by solutions of the dynamic edge effect type

$$v(\xi_1, \xi_2) = V(\xi_1) \sin k_2 (\xi_2 - \xi_2^\circ), \quad \psi(\xi_1, \xi_2) = \Psi(\xi_1) \sin k_2 (\xi_2 - \xi_2^\circ) \qquad (1.4)$$

After substituting (1.4) into (1.1) we obtain a system of ordinary differential equations in the functions $V(\xi_1)$ and $\Psi(\xi_1)$. The characteristic equation of this system has two obvious roots $\lambda = \pm ik_1$, which correspond to the generating solution (1.2). After eliminating these roots, we arrive at the following equation

$$a_0\lambda^6 - a_1\lambda^4 + a_2\lambda^2 - a_3 = 0 \qquad (1.5)$$

Here

$$a_0 = d_{11}, \quad a_1 = 2(b_{12}d_{11} + d_{12})k_2^2 + d_{11}k_1^2 \qquad (1.6)$$
$$a_2 = (b_{11}d_{11} + 4b_{12}d_{12})k_2^4 + 1 + 2b_{12}d_{11}k_1^2k_2^2 - (k_1^2 + \chi k_2^2)^2 f$$
$$a_3 = 2b_{11}d_{12}k_2^6 + 2\chi k_2^2 + b_{11}d_{11}k_1^2k_2^4 + k_1^2 - (k_1^2 + \chi k_2^2)^2 (k_1^2 + 2b_{12}k_2^2) f$$
$$f = (k_1^4 + 2b_{12}k_1^2k_2^2 + b_{11}k_2^4)^{-1}$$

Under homogeneous boundary conditions, a correcting solution of the dynamic edge effect type can always be constructed if three roots of (1.5) have negative real parts [1].

In cases when the correcting solution cannot possibly be constructed, we speak of degeneration of the dynamic edge effect at the appropriate edge. Analogous computations for the edge $\xi_2 = 0$ result in the following expressions for the coefficients of (1.5)

$$a_0 = b_{11}d_{11}^{-1}, \quad a_1 = 2(b_{11}d_{12} + b_{12}d_{11}^{-1})k_1^2 + b_{11}d_{11}^{-1}k_2^2 \qquad (1.7)$$

$$a_2 = (d_{11}^{-1} + 4b_{12}d_{12})k_1^4 + 2b_{12}d_{11}^{-1}k_1^2k_2^2 + \chi^2 - b_{11}(k_1^2 + \chi k_2^2)^2 f$$

$$a_3 = 2d_{12}k_1^6 + d_{11}^{-1}k_1^4k_2^2 + 2\chi k_1^2 + \chi^2 k_2^2 - (b_{11}k_2^2 + 2b_{12}k_1^2)(k_1^2 + \chi k_2^2)^2 f$$

The dynamic edge effect in orthotropic shells will be degenerate if degeneration occurs at least at one of the edges.

We set $s = -\lambda^2$. We then obtain from (1.5)

$$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0 \qquad (1.8)$$

In order for solutions of the dynamic edge effect type to exist, it is necessary and sufficient that (1.8) have no positive roots. As is seen from (1.6), (1.7), the coefficients a_0 and a_1 are nonnegative. For $a_3 < 0$ Eq. (1.8) has one positive root independently of the sign of a_2. This follows from the theorem of Descartes. Therefore, the condition $a_3 > 0$ is necessary for no positive roots. Let us show that this condition is also sufficient. To do this, it is sufficient to show that (1.8) has two complex-conjugate roots for $a_3 > 0$ and $a_2 < 0$ (upon compliance with these conditions there is one negative root). This latter will hold if $D > 0$, where D is the discriminant of the polynomial (1.8). It is easy to see that we have $D < 0$ for $a_3 = 0$ and $a_2 < 0$. Therefore, the boundary of the domain of dynamic edge effect degeneration is determined by the equation $a_3 = 0$.

If we go over to polar coordinates $k_1 = r\cos\varphi$, $k_2 = r\sin\varphi$ $(0 \leqslant \varphi \leqslant \pi/2)$ in the plane of the wave numbers, then the equation of the boundary of the degeneration domain becomes for the edges $\xi_1 = 0$ and $\xi_2 = 0$, respectively

$$r = \left[\frac{(\chi^2 - b_{11})\cos^2\varphi + 2\chi(\chi b_{12} - b_{11})\sin^2\varphi}{(b_{11}d_{11}\cos^2\varphi + 2b_{11}d_{12}\sin^2\varphi)F(\varphi)}\right]^{1/4}, \quad \xi_1 = 0$$

$$r = \left[\frac{(b_{11} - \chi^2)\sin^2\varphi + 2(b_{12} - \chi)\cos^2\varphi}{(d_{11}^{-1}\sin^2\varphi + 2d_{12}\cos^2\varphi)F(\varphi)}\right]^{1/4}, \quad \xi_2 = 0$$

$$F(\varphi) = \cos^4\varphi + 2b_{12}\sin^2\varphi\cos^2\varphi + b_{11}\sin^4\varphi$$

We hence obtain the corresponding conditions for existence of an edge effect

$$(\chi^2 - b_{11})\cos^2\varphi + 2\chi(\chi b_{12} - b_{11})\sin^2\varphi \leqslant 0, \quad \xi_1 = 0$$

$$(b_{11} - \chi^2)\sin^2\varphi + 2(b_{12} - \chi)\cos^2\varphi \leqslant 0, \quad \xi_2 = 0$$

It is seen that the existence of a dynamic edge effect and its degeneration depend only on the ratio between the principal curvatures and the ratio between the compliance matrix elements under membrane deformation. However, the degeneration domains depend on the relationships between the elements of cylindrical stiffness matrix. As in the case of isotropic shells [1], the dynamic edge effect can be oscillatory and nonoscillatory.

Shown in Fig. 1 are the degeneration domains for the dynamic edge effect for different values of χ for $E_1/E_2 = 0.1$, $\nu_2 = 0.3$, $\gamma = (1 + \nu_2)^{-1}/2$ (a) $E_1/E_2 = 10$, $\nu_1 = 0.3$, $\gamma = (1 + \nu_1)^{-1}/2$ (b). The edge effect at the edge with greatest curvature $(\xi_1 = 0)$ is nondegenerate for a shell of nonnegative Gaussian curvature if $0 \leqslant \chi \leqslant b_{11}/b_{12}$. For $E_1/E_2 > 1$ we can have $b_{11}/b_{12} < 1$, and in this case degeneration of the dynamic edge effect is possible. Let us note that the edge effect in the case of isotropic shells is always nondegenerate.

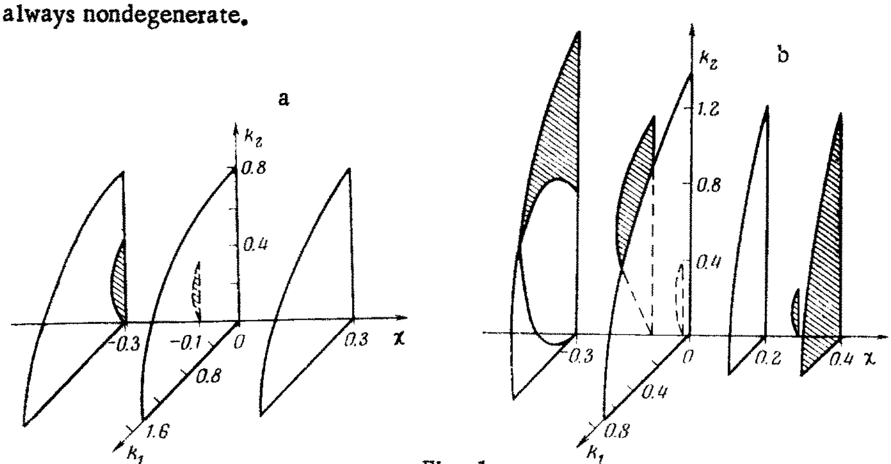

Fig. 1

Another singularity inherent in orthotropic shells of nonnegative Gaussian curvature, appears at the edge with lesser curvature ($\xi_2 = 0$). The edge effect is not degenerate for $\chi \geqslant (E_2 / E_1)^{1/2}$. If $E_2 / E_1 < 1$, then χ are found such that the edge effect will be nondegenerate. Therefore, for $E_1 / E_2 > 1$ there exist values of χ for which the edge effect will be nondegenerate along any of the principal directions.

In this sense, the range of application of the asymptotic method to orthotropic shells is broader than for isotropic shells of corresponding geometry.

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PARAMETRIC VIBRATIONS OF A VISCOELASTIC BAR WITH NONLINEAR HEREDITARY CHARACTERISTIC

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The dynamic stability and resonance modes for parametric vibrations of a viscoelastic bar subjected to a harmonic force are investigated by the method of averaging [1−3]. The connection between the stress and strain is given as the sum of multiple integrals [4−6].